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cal thought and skill, and they will be lasting monuments to his memory. His style is direct, clear, and elegant. His solutions are neat, accurate, and simple. He has that rare and happy faculty of presenting his solutions in the simplest mathematical language, so that those who have mastered the elements of the various branches of mathematics, are able to understand his reasoning.

Dr. Martin is now (1894) editor of the *Mathematical Magazine*, and the *Mathematical Visitor*, two of the best mathematical periodicals published in America. These are handsomely arranged and profusely illustrated with very beautiful diagrams to the solutions,he doing the typesetting with his own hand. The typographical work of these journals is said to be the finest in America. The best mathematicians from all over the world contribute to these two journals. The *Mathematical Visitor* is devoted to Higher Mathematics, while the *Mathematical Magazine* is devoted to the solutions of problems of a more elementary nature. All solutions sent to Dr. Martin receive due credit, and, if it is possible to find room for them, the solutions are all published. He has thus encouraged many young aspirants to higher fields of mental activity. He is always ready to aid any one who is laboring to bring success with his work. He is of a kind and noble disposition and his generous nature is in full sympathy with every diligent student who is rising to planes of honor and distinction by self application and against adverse circumstances.

Dr. Martin has a large and valuable mathematical library containing many rare and interesting works. His collection of American arithmetics and algebras is one of the largest private collections of the kind in this country. He has also a large collection of foreign arithmetics and algebras; a large collection of grammars of the English language, mostly American; and a large collection of miscellaneous works, including botany, natural history, biography, poetry, American dictionaries, &c., and many curious books.

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LOGARITHMS OF NEGATIVE NUMBERS.

By **M. C. STEVENS**, Professor of Higher Mathematics, Perdue University, Lafayette, Indiana.

On page 28, of B. O. Peirce's "Short Table of Integrals" is the following:—Log $(-u)=\log u+$ a constant. From the ordinary treatises on logarithms we learn that all possible numbers from $-\infty$ to $+\infty$ are required to express the logarithms of numbers from 0 to $+\infty$. The thoughtful student will therefore enquire, what is the nature of the constant which is added to the ordinary logarithm of u to give the log $(-u)$. The following explanation, although it is not new, may be interesting to some of the readers of the MONTHLY. I therefore submit it.

It is shown in many treatises on Higher Algebra, (See Wentworth's

College Alg. p. 492)that $e^{xi}=\cos x+i\ \sin x$, (i being used for the square root of -1). Since the trigonometric functions of $x+2a\pi$ are the same as for x (a being any integer) it follows that $e^{(x+2a\pi)i}=\cos(x+2a\pi)+i\sin(x+2a\pi)$....(1). Making $x=\pi$, we have $e^{\pi(1+2a)i}=-1$. $\therefore$ $\log(-1)=\pi(1+2a)i$.....(2).

If in equation (1) we make $x=0$ there results $e^{(2a\pi)i}=1$.

$\therefore$ $\log 1=(2a\pi)i$.....(3). If now we put $u=e^x$, then the ordinary $\log u=x$. But $-u=e^x.(-1)=e^{x+\pi(1+2a)i}$.

Whence, the general $\log(-u)=x+\pi(1+2a)i=$ *ordinary* $\log u+\pi(1+2a)i=\log u+\log(-1)$.

Thus it is shown that the constant in question is the impossible expression, $\pi(1+2a)i$, which by equation (2) is $\log(-1)$.

Again $u=e^x=e^x.(1)=e^{x+(2a\pi)i}$.

$\therefore$ It may be shown that the *general* $\log u=$*ordinary* $\log u+(2a\pi)i$.

Hence, as a can have any integral value, it appears that any number, either positive or negative, can have an infinite number of logarithms; but only positive numbers can have their logarithms expressed by arithmetical numbers, and then only when $a=0$.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph.D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER FIRST.

[Continued from the March Number.]

EUCLIDEANS.

John Wallis(1616—1703) deduces Euclid's Parallel-Postulate from the assumption of two similar figures, for example, two unequal triangles having their angles respectively equal and their sides proportional.

But we know that there is no need of assuming so much. The Parallel-Postulate follows from the existence of two unequal triangles of equal angle-sum,without regard to the proportionality of the sides or the respective equality of the angles.

This remarkable truth was established in the very first Non-Euclidean Geometry published in 1733. After this date there existed in the world not only Euclid's, but also the two Non-Euclidean geometries; and so henceforth